Pinwheel-like structures resulting from interaction of plane pulses of excitation

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We demonstrate that complex spatiotemporal structures may appear in an excitable system as the result of interaction between two plane pulses. Such behavior has been obtained for FitzHugh-Nagumo type of dynamics by numerical integration of reaction-diffusion equations.

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I. INTRODUCTION

The reactors composed of active regions, in which reactions occur, and passive areas, where some of the reagents are absent and so only a part of reactions proceed there have been recently intensively studied [1,2]. In practical applications such reactors can be realized using an immobilized catalyst which is inhomogeneously distributed in space and passive areas are those which do not contain it. The interest in such reactors comes from the fact that they may be applied in direct processing of chemical signals. One can consider coding information with the use of chemical systems by assigning a high concentration of a selected reagent with the logical ‘‘true’’ state and a low concentration with the logical ‘‘false.’’ A pulse of concentration of a selected reagent, which can propagate in an excitable system, may be regarded as a chemical signal. One can construct reactors which process such chemical signals, such as signal diodes [3], logical gates [1], memory cells [2], and even counters of number of pulses [4,5]. In all these devices passive barriers separating excitable areas play an important role. Here we show that two pulses of excitation interacting over a barrier may create an interesting spatiotemporal pinwheel-like structure.

II. THE FITZHUGH-NAGUMO MODEL

The FitzHugh-Nagumo (FHN) model of an excitable dynamics was originally introduced to describe the behavior of nerve tissues [6,7]. Here we use its yet simplified version, proposed by Yoshikawa, Motoike, and Kajiya in their study on information processing in chemical systems [8,1]. The dynamics in active areas is described by the following equations [1,6,7]:

$$\tau \frac{\partial u}{\partial t} = -\gamma[ku(u-\alpha)(u-1)+v] + D_u \nabla^2 u, \qquad (1)$$

$$\frac{\partial v}{\partial t} = \gamma u, \qquad (2)$$

with the parameters $\tau=0.03$, $\gamma=1$, $k=3.0$, $\alpha=0.02$ (as given by Motoike and Yoshikawa in [1]) and $D_u=0.00045$ [9]. For these values of parameters the system has a single stationary solution $(u,v)=(0,0)$, homogeneous in space, which is excitable. The system may be excited by a local decrease in the value of v, which initiates a propagating pulse. The variables u and v cannot be directly associated to concentrations of chemical spices, but their behavior resembles the one of the activator (u) and inhibitor (v) in a chemical system.

We assume that in the passive areas the kinetic terms are absent in the corresponding equations. The diffusion of activator is possible, thus it is natural to call these regions ‘‘diffusion areas.’’ The equations describing the time evolution of u and v in these areas are [1]

$$\tau \frac{\partial u}{\partial t} = D_u \nabla^2 u, \qquad (3)$$

$$\frac{\partial v}{\partial t} = 0 = \text{const.}, \qquad (4)$$

with $\tau=0.03$ and $D_u=0.00045$, as in the excitable areas. Of course $u=v=0$ is also a stationary homogeneous solution of Eqs. (3) and (4). Thus, the stationary values of u and v in both the active and passive media are equal to 0, but in the active part this solution is excitable. The system of equations (1)–(4) has been used in studies on logical gates for chemical signals [1], chemical diode [1] and on a switch of a chemical signal direction [9].

In [1] Motoike and Yoshikawa discussed the problem of excitation of an active area by a pulse propagating in another active area, when both areas are separated by a passive stripe. Of course, a pulse in one active area may excite the other active area if the passive stripe is narrow. The maximum width of the passive stripe for which such excitation still occurs is called the penetration depth. It has been found [1] that the penetration depth depends on the geometry of the junction and on the direction of propagation of incident pulses and it is maximal for plain pulses traveling in the direction perpendicular to the barrier. In [1] Motoike and Yoshikawa studied the FitzHugh-Nagumo model with the same values of parameters as given above and they found that if the penetration depth for a single pulse with the wave vector perpendicular to the barrier is denoted by d_c, then the penetration depth for a pulse traveling parallel to the barrier it is only $0.94\cdot d_c$. In the considered units $d_c \approx 0.163$ [1,10]. Therefore, it is possible to adjust the width of the passive

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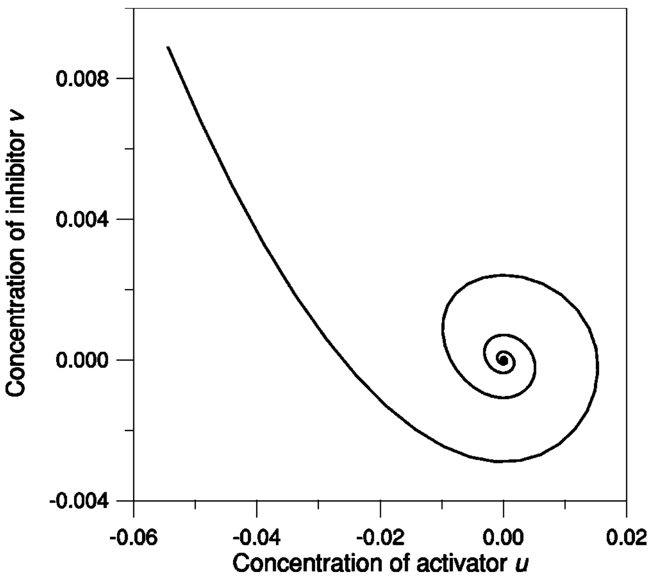

FIG. 1. The evolution towards the stable state for the considered FitzHugh-Nagumo model. Values of u and v are given in dimensionless units.

layer d in such a way that it is semi penetrable, i.e., it is transparent for a pulse propagating perpendicularly, but impenetrable for a pulse propagating parallel to it. In the units used, d should satisfy $0.94\cdot d_c < d < d_c$.

Calculations for the FitzHugh-Nagumo model have shown that when a regular train of excitable pulses arrives at the passive barrier then some pulses from the train are able to cross a passive barrier, which is wider than the penetration depth for a single pulse (d_c) [10–12]. The mechanism of this phenomenon is the following. Figure 1 shows the relaxation towards the stable state on the phase plane $u\times v$. The curve plots the dependence $v(u)$ close to the stable state and in this region $v(u)$ is almost the same for exciting perturbations as well as for nonexciting perturbations with sufficiently large amplitudes. It is important that for the FitzHugh-Nagumo model the stable state is reached through dumped oscillations. Let us assume that a barrier is not penetrated by the first pulse. The system behind the barrier is not excited, but it is still perturbed and relaxes as shown in Fig. 1. If the second perturbation comes it may find the system behind the barrier in a state characterized by a positive value of activator and a negative value of inhibitor. And such a state may be excited by a smaller perturbation than the one needed to excite the stationary state. As a result the second (or later) pulse may cross a barrier which is nonpenetrable for the first one. Such a phenomenon is absent in the models of Belousov-Zhabotinsky reaction [13] (the Rovinsky-Zhabotinsky model [14–17] and the Oregonator model [18–20]) considered in [5,10,11]. In those models there are no oscillations around the stable state. Moreover, the mechanism of activator's decay is present in the passive areas and it is more efficient than in the active ones. As the result, the barrier which is transparent for a single pulse may be impenetrable for the train of pulses.

III. COMPLEX STRUCTURES RESULTING FROM INTERACTIONS OF TWO PULSES

Let us consider two excitable areas [within which Eqs. (1) and (2) hold], separated by a semi-penetrable stripe of the passive medium [Eqs. (3) and (4)]. We selected its width as $d=0.16$, so the barrier is transparent for pulses propagating perpendicularly, but impenetrable for those propagating parallel to it. We will show that in such systems very interesting spatiotemporal structures may emerge, if two excitable pulses interact via such a passive barrier.

The interaction of pulses is investigated by numerical integration of the reaction-diffusion equations (1)–(4). We studied an area 8.0×8.0 units with a semipenetrable passive stripe in the middle. It is covered with a square grid of 400×400 points. For this grid the passive stripe was 8 grid points wide. We assumed that a free flow of the activator is possible between the active (excitable) and the passive fields and that there are no flux boundary conditions on the borders of the square. The integration was carried out with an implicit method based on the Crank-Nicolson discretization of the Laplace operator [21]. We used the time integration step $dt=0.005$. At the beginning the values of u and v in both active and passive areas corresponded to the stationary states $(u=v=0)$. Pulses in the investigated system were initiated by local decreasing v to $v_{ini}=-0.2$. The calculations were carried out up to $t_{max}=200$.

Figure 2 presents the evolution of the system with two plane pulses of excitation, traveling along the passive stripe in opposite directions. In Fig. 2, the gray areas show the excitable field, the black horizontal line marks the diffusion stripe, and lighter areas correspond to higher values of u. The pulse located in the bottom active area travels to the right, while the other one, in the upper active area, moves to the left [Fig. 2(a)]. At a certain time the pulses go past each other [Fig. 2(b)]. The region after each pulse relaxes towards the stable state via oscillations and at a certain point the perturbation generated by a pulse propagating on the other side of the barrier appears to be sufficiently large to excite it. The excitations appear symmetrically on both sites of the barrier [Fig. 2(c)] and circular pulses develop [Fig. 2(d)]. And yet again, there are two excitable pulses which move on both sides of the passive stripe in opposite directions; they meet [Fig. 2(e)] and the whole scenario repeats [Figs. 2(f), 2(g), and 2(h)]. The stable pinwheel-like structure is formed.

Another interesting, but qualitatively similar, type of evolution is observed for pulses traveling in perpendicular directions (Fig. 3). One of the pulses, located in the upper part of the system, moves along the passive stripe, to the right. If it is alone, it never crosses the barrier. Another pulse, located in the bottom part of the system, travels upwards, in the direction perpendicular to the stripe [cf. Fig. 3(a)]. The pulse initially located in the bottom part of the system excites the area behind the barrier almost everywhere except a narrow interval behind the upper pulse where the medium is not relaxed yet. As a result, we obtain a pulse spreading to the right (following the first pulse) and upwards [Fig. 3(b)]. On its way to the right it propagates along the passive stripe and introduces perturbation into the active area below the stripe.

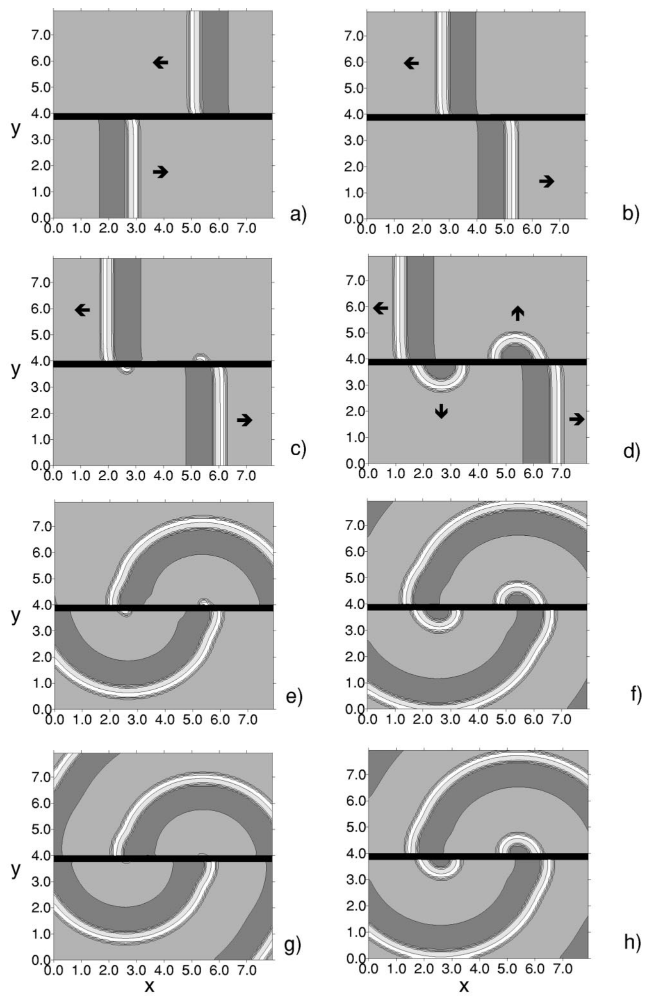

FIG. 2. The "pinwheel" on a plane with a single stripe of diffusion field, occurring when two excitable pulses traveling in perpendicular directions meet. The consecutive snapshots present the contours of u as a function of space coordinates (x,y) (white areas, $u>0.4$; light grey areas, $-0.2<u<0.1$; dark grey areas, $u<-0.2$) at moments of time: (a) $t=4.00$, (b) $t=7.00$, (c) $t=8.00$, (d) $t=9.00$, (e) $t=12.00$, (f) $t=17.00$, (g) $t=20.00$, and (h) $t=21.00$. The black horizontal line marks the position of passive stripe; all remaining areas are active. The distances on X and Y axis are given in dimensionless units of distance.

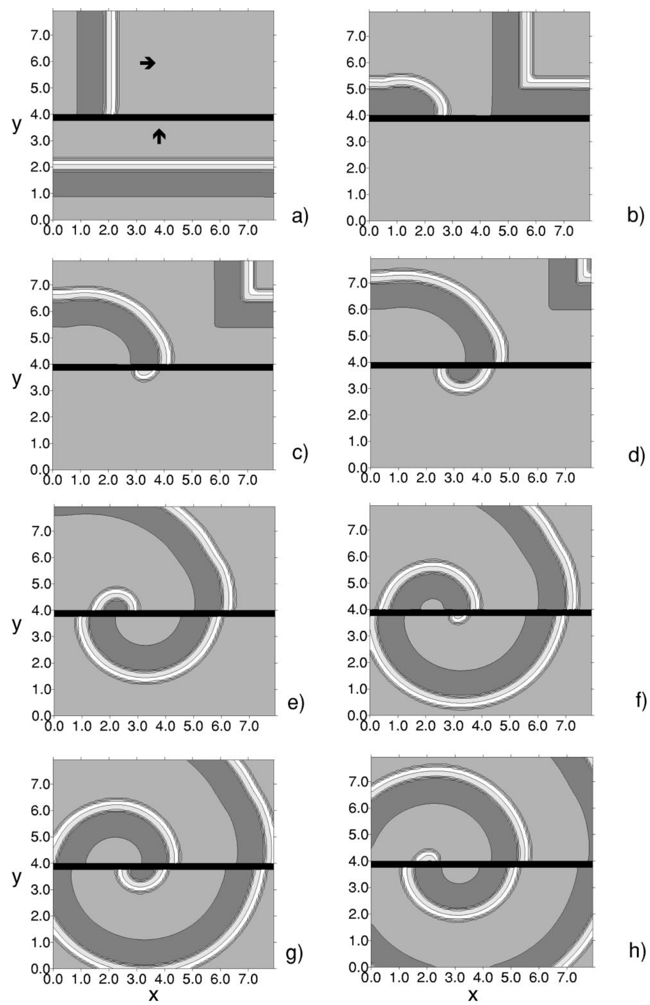

FIG. 3. The "pinwheel" on a plane with a single stripe of diffusion field, occurring when two excitable pulses traveling in perpendicular directions meet. The consecutive snapshots present the contours of u as a function of space coordinates (x,y) at moments of time: (a) $t=3.00$, (b) $t=7.50$, (c) $t=9.25$, (d) $t=10.00$, (e) $t=12.00$, (f) $t=13.25$, (g) $t=14.00$, and (h) $t=15.50$. The gray scale is the same as in Fig. 2.

If this perturbation matches with the state of the system in the area below then the excitation appears [Fig. 3(c)]. A circular pulse is created [Fig. 3(d)] and one branch of it follows the pulse on the upper side. It does not excite the upper active part because the medium is not relaxed yet. The other part, propagating to the left, finds its way from the barrier toward the relaxed upper active medium and creates a circular pulse on the other side of it [Fig. 3(e)]. It expands and creates a pulse which crosses the barrier again [Figs. 3(f)–3(h)].

Numerically both structures presented in Figs. 2 and 3 are stable. They persisted in calculations lasting twenty times longer than a pulse needed to get through the whole square. We did not observe any change in the position of the center of the pinwheel in time. In order to test the numerical stability with respect to parameters of integration we repated calculations using 800×800 grid covering the same area, so the spacing between grid points was reduced by half. The passive stripe was 17 grid points wide. The results were the same as shown in Figs. 2 and 3. We performed similar calculations for systems with the passive stripe wider or narrower by 1% with respect to the one used in Figs. 2 and 3. Comparing with the evolution presented in Figs. 2 and 3, we have not found any remarkable differences in the appearance, shape, or stability of the pinwheels obtained from these calculations. However, the range of barrier's widths in which the pinwheels are observed is narrow. We have also performed calculations for the passive stripe by 10% wider and such a barrier is too wide to be crossed. The pulses shown in Fig. 2 propagate without visible interactions. In the case considered on Fig. 3 the pulse propagating upwards dies at the barrier without affecting the pulse propagating in the upper active area. If the barrier is by 5% narrower than the one considered on Figs. 2 and 3 then it can be penetrated by an inpulse propagating parallel to it. As a result the excitation spreads quickly in the whole system and the system returns to the stationary state without forming any stable spatial structure.

IV. CONCLUSIONS

In the paper we have discussed interesting examples of pinwheel-like spatiotemporal structures in two-dimensional systems composed of active and passive areas in which dynamics is described with the FitzHugh-Nagumo type model [Eqs. (1)–(4)]. These structures are numerically stable and they are observed in a large range of widths of semipenetrable passive stripes.

The "pinwheels" would definitely influence signal processing in reactors with very wide signal channels. One of the reactors which may be affected is a cross junction coincidence detector [9,17] in which the pinwheel appears when one of the signals follows just after another. A pinwheel may also block the negation gate described in [1]. Therefore their existence should be taken into account when signal processing reactors are designed.

We have repeated similar calculations using the Rovinsky-Zhabotinsky (RZ) model of the ferroin catalyzed Belousov-Zhabotinsky reaction [14–16] and the same values of its parameters as in [17]. We have not observed "pinwheels" in the calculations based on the RZ model because of a fast relaxation of activator inside a barrier and the absence of oscillations when the system approaches the stable state.

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